Modelling of Energy Dissipation during Transient Flow

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***Abstract*— An important issue in the context of design and analysis of a piping system is the rate of energy dissipation during transient flow. The hyperbolic classical water hammer equations do not account the damping and attenuation of laminar and turbulent fluid transients in the piping systems. In this paper, a classical transient model is developed, which simulates the transient condition during valve closure in a simple reservoir-steel pipe-valve system. The developed base model was modified again by the addition of convective term. The obtained results were then compared with results from the Surge2000 software, for validation and found in good agreement. However, the experimental results show large damping of pressure waves after the first peak, indicating significant energy dissipation. In order to account the energy dissipation, the governing equations were further modified to incorporate variable wave celerity into the continuity equation and the diffusive term into the momentum equation. The resulting dispersive error was then reduced by considering artificial viscosity, which further improves the computed results close to the experimental results.**

Keywords*— dissipation; convective; damping; diffusive; dispersive; viscosity

I. INTRODUCTION

Hydraulic transients are critical design factors in a large number of fluid systems from automotive fuel injection to water supply, transmission, and distribution systems. Today, long pipelines transporting fluids over great distances are so common, and the high-velocity water distribution systems are also increased. Mechanically sophisticated fluid control devices, including many types of pumps and valves, coupled with increasingly sophisticated electronic sensors and controls, provide the potential for complex system behaviour. Huge amount of money has been spent for such complex systems.

In any complex piping system, always, there is a need to adjust the flow continously for which the operating conditions of valves and pumps are to be changed. Transient flow occurs in a system whenever the flow in the system is suddenly changed. It is always accompanied by high or low pressure peaks. Pipe system designers assume that transients tend to decay slowly and the computation of transient analysis is carried out on the basis of classical water hammer equations. However, discrepancy is observed between the actual and computed pressures. The difference between the observed and calculated pressure characteristics during dynamic processes is called energy dissipation [9].

It has been found in the recent researches that transient analysis overdesigns the piping systems [7]. The pipe wall elastic behaviour damps the hydraulic transient much faster than that computed results. Thus, the importance of wall friction and energy losses during transient has lead to the development of an appropriate model for energy dissipation [8]. The main objective of this study is to create an appropriate model for energy dissipation during transients and calibrate it with experimental data. Stages of study are:

1. Creating a numerical model in MATLAB for the classical continuity and momentum equations.
2. Validating the results using Surge2000 for the transient analysis.
3. Incorporating the variable wave celerity in the continuity equation and diffusive term in the momentum equation and observe its relevance in predicting the wave attenuation.
4. Modifying the existing model to reduce the dispersive errors.

II. CREATION OF CLASSICAL TRANSIENT MODEL

The classical water hammer equations for analysing transients in piping systems is given by (1) and (2) [6].

$$\frac{\partial H}{\partial t}+\frac{c^2}{gA}\frac{\partial Q}{\partial x}=0 \tag{1}$$

$$\frac{\partial Q}{\partial t}+gA\frac{\partial H}{\partial x}+\frac{f}{2DA}Q|Q|=0 \tag{2}$$

where Q- Fluid discharge (cumecs), H-Pressure head of the pipe(m), f- Darcy's Weisbach friction factor, A-Cross sectional area of pipe (m^2), g=Acceleration due to gravity (m/s^2), c=Wave celerity (m/s), D=Pipe diameter (m), t=Time (seconds), x=Abscissa along the centre line of the pipe (m).

A. Solution of Classical Water hammer equations

Governing equations for representing transient flow in pipes (continuity and momentum) are quasi-linear, hyperbolic, partial differential equations. Several numerical methods are used for solution of these equations. Among these methods, finite difference methods have been utilized very extensively. The numerical solutions, based on the finite differences, provide us with the values of the dependent variables at discrete points which are known as grid points. The present study uses an explicit scheme-MacCormack approach- for the solution of these partial differential equations, because of its simplicity and easiness in programming [14]. MacCormack method consists of two steps; a *predictor* step which is followed by a *corrector* step [3]:

1. Forward difference *predictor* step followed by backward difference *corrector* (on odd time intervals).
2. Backward difference *predictor* step followed by forward difference *corrector* (on even time intervals).

III. MODIFICATION OF CLASSICAL TRANSIENT MODEL BY THE ADDITION OF CONVECTIVE TERMS

The classical transient model was modified to include the convective terms. While deriving the continuity and momentum equations using the basic theory of conservation of mass and energy, the convective terms were neglected as its value was small and facilitates the solution using the method of characteristics [11]. In order to represent complete dissipation phenomena, the convective terms are included in the classical set of equations and thus the governing equations are rewritten as (3) and (4).

$$\frac{\partial H}{\partial t} + \frac{Q}{A}\frac{\partial H}{\partial x} + \frac{c^2}{gA}\frac{\partial Q}{\partial x} = 0 \qquad (3)$$

$$\frac{\partial Q}{\partial t} + \frac{Q}{A}\frac{\partial Q}{\partial x} + gA\frac{\partial H}{\partial x} + \frac{f}{2DA}Q|Q| = 0 \qquad (4)$$

IV. Incorporation of Variable Wave Celerity into the Continuity Equation

For a given pipe diameter and liquid, the pressure wave celerity is constant and is calculated using (5).

$$C=\frac{\sqrt{\frac{K}{\rho}}}{\sqrt{1+\frac{K.D}{E.e}}} \qquad (5)$$

where K= Bulk modulus of elasticity of the fluid (Pa), ρ = Density of the fluid (kg/m^3), D=Diameter of the pipe (m), e=Thickness of the pipe wall (m), E= Young's modulus of elasticity of the pipe wall material.

But when transient occur the liquid and the pipe wall will be prone to compression. Thus, pipe diameter and cross sectional area will be changing with time which will in turn alter the wave speed. According to Mitosek and Symkiewicz [11], in every cycle of head variation, one can distinguish four segments having various curvatures. This variation can be related to the pressure wave phase. In one wave cycle, there are four time intervals of different lengths. Generally, it is observed that the time of the pressure increase is shorter than the time of the pressure decrease. This difference seems to be related to the pressure-wave amplitude, because it disappears with time as the amplitude decreases. Although these intervals have different lengths, their sum, being the total wave period, is constant with time. It is evident that the asymmetry observed in head variation must be related to the pressure wave celerity. Therefore instead of considering 'c' as a constant, c(x, t) should be used [11].

Taking into account the varying gradients of function head variation one finds that in the phase of compression, when the stream of liquid slows down, the wave travels at a speed less than the average one [c(x, t) < c], whereas in the phase of decompression, when the stream accelerates, the wave travels at a speed greater than the average one [c(x, t) > c] [11].

In the derivation of classical Joukovsky (Korteweg) formula (6), perfect elastic behaviour of the liquid and pipe material was assumed [15].

$$\Delta p = U\rho c \qquad (6)$$

where Δp is the increment of pressure, c is the wave speed, ρ is the density of fluid and U is the velocity.

If real bodies are considered, during the compression of liquid in the pipe due to immediate valve closure, a fraction of energy, denoted as E_{diss}, is lost because of dissipation [4]. Therefore, as per the principle of conservation of energy, kinetic energy of the flowing water, E_{kin} turns into the elastic energy of the water, E_l, and the elastic energy of the pipe material, Ew and the energy lost due to dissipation, E_{diss}.

$$E_{kin}=E_{diss}+E_l+E_w \qquad (7)$$

where

$$E_{kin}=\rho g A\,\Delta x\frac{U^2}{2g} \qquad (8)$$

$$E_l=\Delta p^2\,\frac{A.\Delta x}{2K} \qquad (9)$$

$$E_w=\Delta p^2\,A\,\frac{D.\Delta x}{2E.e} \qquad (10)$$

Assuming that the dissipated energy constitutes a part of the elastic energy of the liquid and the pipe wall, this fraction can be denoted using the coefficient β.

$$\beta=\frac{E_{diss}}{E_l+E_W} \qquad (11)$$

In the phase of decompression, the energy stored due to the elasticity of the liquid and pipe material is reconverted into the kinetic energy of the flowing water and the E_{diss} term becomes negative and in turn β becomes negative.

For preliminary evaluation of the modified continuity equation, a very simple algorithm for the wave celerity correction is assumed, namely, taking into account the value of the current pressure H(t) and the value of the pressure gradient $\frac{\partial H}{\partial t}$, the wave period is divided into four parts. The applied correction process is carried out as follows:

1. If $H(x,t)>H(x,0)$ and $\frac{\partial H(x,t)}{\partial t}<0$, then $c(x,t)=\frac{c}{\sqrt{1-\beta 1}}$.
2. If $H(x,t)<H(x,0)$ and $\frac{\partial H(x,t)}{\partial t}<0$, then $c(x,t)=\frac{c}{\sqrt{1+\beta 1}}$.
3. If $H(x,t)<H(x,0)$ and $\frac{\partial H(x,t)}{\partial t}>0$, then $c(x,t)=\frac{c}{\sqrt{1-\beta 1}}$.
4. If $H(x, t)>H(x, 0)$ and $\frac{\partial H(x,t)}{\partial t}>0$, then $c(x,t)=\frac{c}{\sqrt{1+\beta 1}}$.

where H(x,0) is the hydrostatic pressure in the pipe when the state of rest is reached, whereas c is the standard wave celerity. The above presented formulas hold for advancing time and for $0\le x\le L$. Thus, variable wave celerity was defined for varying pipe length and time. Thus, the continuity equation is modified by incorporating variable wave celerity instead of the constant wave celerity.

$$\frac{\partial H}{\partial t}+\frac{Q}{A}\frac{\partial H}{\partial x}+\frac{[c(x,t)]^2}{gA}\frac{\partial Q}{\partial x}=0 \quad (12)$$

V. INCORPORATION OF DIFFUSIVE TERM INTO THE MOMENTUM EQUATION

By adjusting the values of numerical parameters such as the time step, Δt, the space interval, Δx, or the weighting parameters, it is possible to obtain a remarkably better match of the results of computation and observation. This well-known effect is related to the numerical diffusion generated by the applied method of solution. The positive role of the numerical diffusion has suggested a way of improvement of the solution algorithm for some problems described by the hyperbolic equations. Such an approach, known as the pseudo viscosity method [2], was proposed by von Neumann and Richtmyer [12]. It deals with the introduction of a diffusive term into the energy or dynamic equation. This term, having smoothing properties, provides control the solution near discontinuity when non-dissipative methods are applied. Its particular form relates the intensity of the generated diffusion to the wave steepness, ensuring that this extra term acts strongly only locally, becoming insignificant far away from discontinuity. This extra term will have a coefficient, v^G [11].

$$v^G=v^M+v^T+v^V+v^F \quad (13)$$

where v^M-coefficient of molecular diffusion of momentum, v^T-coefficient of turbulent diffusion of momentum, v^V-coefficient of second viscosity, v^F -coefficient of diffusion related to the dissipation at the front of the pressure wave.

The momentum equation will be then modified into (14) [11].

$$\frac{\partial Q}{\partial t}+\frac{Q}{A}\frac{\partial Q}{\partial x}+gA\frac{\partial H}{\partial x}+\frac{f}{2DA}Q|Q|-\frac{v^G}{A}\frac{\partial^2 Q}{\partial x^2}=0 \quad (14)$$

VI. MODIFICATION OF NUMERICAL MODEL TO REMOVE THE DISPERSIVE ERRORS

In order to remove the dispersive errors in MacCormack scheme, a procedure developed by Chaudhary [5] was used to dampen numerical instabilities. This procedure, smoothes regions of large gradients while leaving smooth areas relatively undisturbed. The values of the variables at the new time computed by MacCormack method are modified using the following algorithm:

$$\vartheta_i=\frac{|H_{i+1}-2H_i+H_{i-1}|}{|H_{i+1}|+2|H_i|+|H_{i-1}|} \quad (15)$$

$$\vartheta_{i+\frac{1}{2}}=\Phi\max(\vartheta_{i+1},\vartheta_i) \quad (16)$$

in which Φ is dissipation constant used to regulate the amount of artificial viscosity. At nodes near to the boundaries a one-side finite difference approximation is used.

$$\vartheta_i=\frac{|H_i+H_{i-1}|}{|H_i|+|H_{i-1}|} \quad (17)$$

$$\vartheta_i=\frac{|H_{i+1}-H_i|}{|H_{i+1}|+|H_i|} \quad (18)$$

The computed dependent variable *f* is then modified as

$$f_i^{\,j+1}=f_i^{\,j+1}+\vartheta_{i+\frac{1}{2}}\left(f_{i+1}^{\,j+1}-f_i^{\,j+1}\right)-\vartheta_{i+\frac{1}{2}}\left(f_i^{\,j+1}-f_{i-1}^{\,j+1}\right) \quad (19)$$

f refer to both the dependent variables, H and Q. The above procedure is equivalent to adding second-order dissipative terms to the original governing equations [1].

VII. NUMERICAL RESULTS

The developed base model was tested for the experiment done by Mitosek and Symkiewicz for the steel pipe [10]. The experimental installation is specified in Fig. 1. It is composed of a straight pipeline (1), pressurized tank (2) and a ball valve (4) mounted at the end of the pipe. The valve was closed manually. In this experimental study, the valve closure time is measured by the time recorder with an accuracy of 0.001s, ranged from 0.018 to 0.025s. The pressure was recorded by means of a measuring system consisting of strain gauges (5), extensometer amplifier (6) and a computer (7) with AD/DA (20MHz) card. The input data for the second trial of the base model is provided in Table 1. The result of the steady state analysis is plotted in Fig. 2.

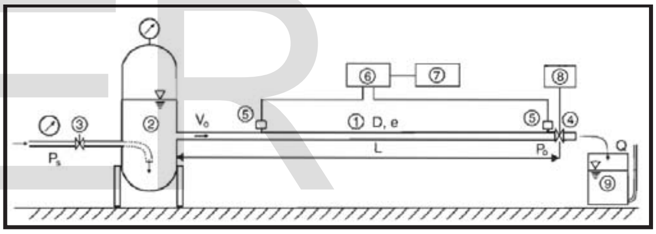

Fig. 1. Experimental Installation [10]

TABLE I. INPUT DATA FOR STEEL PIPE [10]

PARAMETER	VALUE
Length(m)	72
Inside diameter(m)	0.042
Thickness of pipe wall(m)	0.0033
Roughness height (m)	0.00008
Wave velocity (m/s)	1245.0
Initial pressure head (m)	51.00
Initial velocity(m/s)	0.410

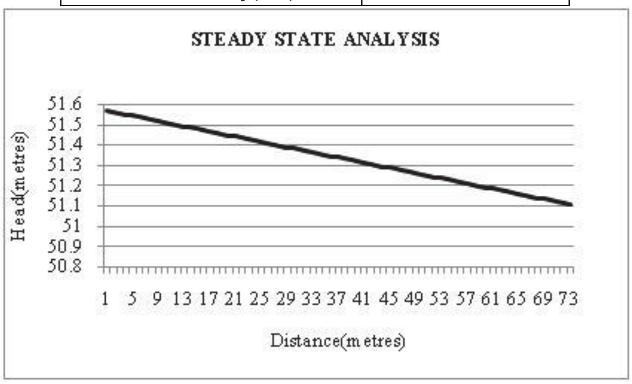

Fig. 2. Steady state analysis for the steel pipe

There is a constant decrease in head from the reservoir till the valve end. The obtained steady state head at valve end is 51.11m.

A. Creating the classical transient model

The classical transient model or the base model was applied to the experimental data for steel pipe [10]. Since the valve used for the experiment was of ball type, the valve characteristic for ball valve was fed to the model (Fig. 3).The valve closure time was taken as 0.021seconds which is the average of the measured time of closure (0.018 to 0.025seconds). The simulation time was taken as 12 seconds. The corresponding head variation at the valve is given in Fig. 4. The peak value of pressure head at the valve closure gradually decreased as the time progresses. However, from the figure the time at which the head attains a stable value cannot be interpreted. Thus, it can be ascertained that the head attains stable value at a time greater than the provided simulation time, i.e., 12 seconds. This result is then compared with that of experimental results and is shown in Fig. 5. Though peak head is matching well at the first crest, drastic difference is observed between computed and experimental results afterwards. The reason for the discrepancy may be either due to the shorter valve closure time (i.e. time of closure < (*L/a*)) or due to the omission of certain phenomena which was actually present in the system. However, a strong damping of pressure head is observed for the experimental result.

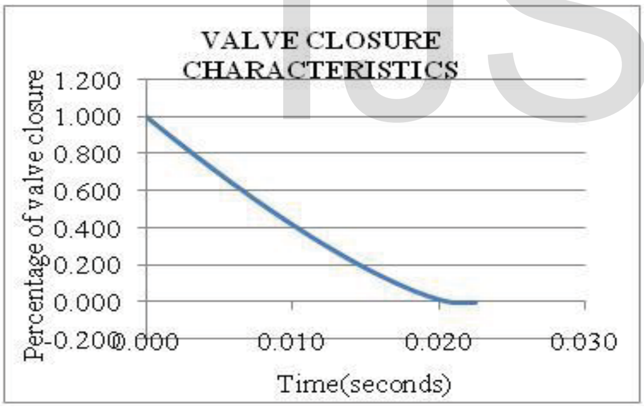

Fig. 3. Valve characteristic curve

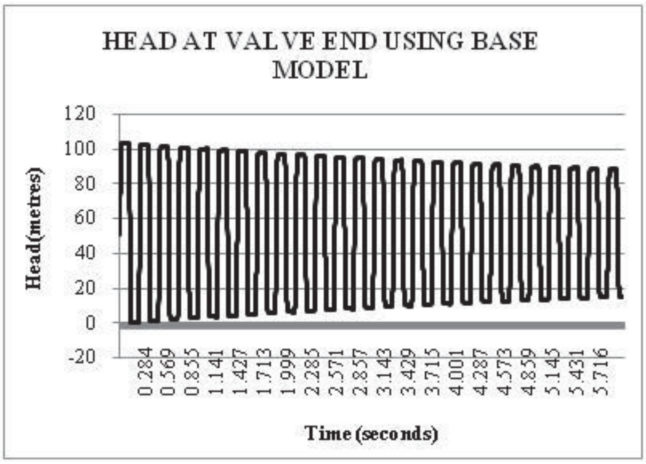

Fig. 4. Computed pressure head at valve for the steel pipe data [10]

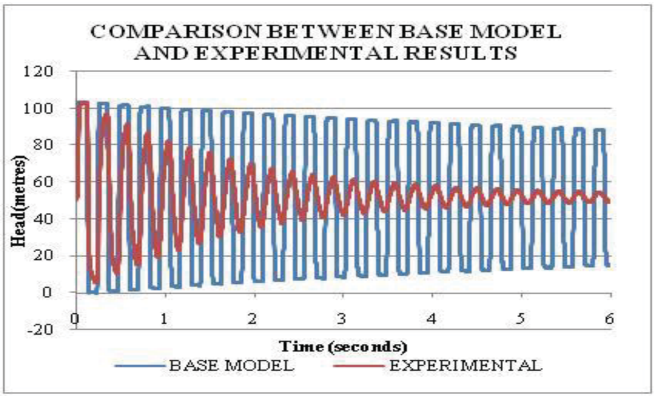

Fig. 5. Comparison between base model and experimental results at valve end

In order to study the discrepancy between the results (Fig. 5), the base model was modified to include the convective terms. Reference [13] reported its applicability (i.e. addition of convective terms) to a wide range of transient flow problems. Transient analysis with the convective term ((3) and (4)) was carried out for the experimental installation provided in Fig. 1. The head variation at the valve end for the modified base model is provided in the Fig. 6. It shows only a marginal reduction upto 0.008%.

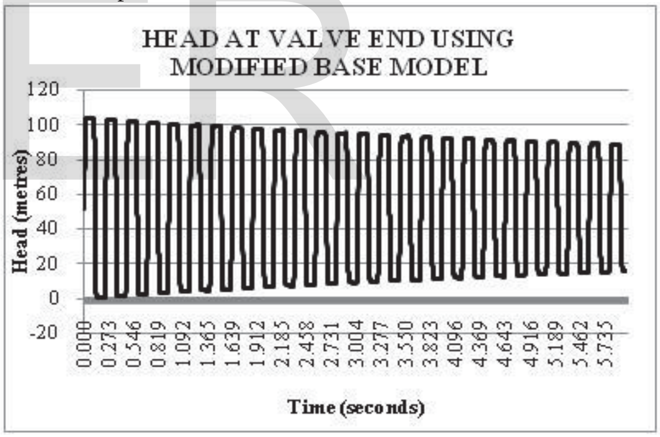

Fig. 6. Head at valve end for steel pipe using modified base model

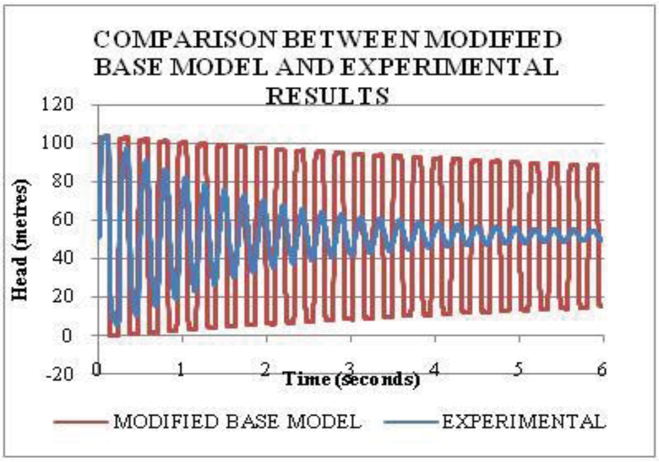

Fig. 7 Comparison between modified base model results and experimental formulation for steel pipe

For evaluating the ability of the modified model to determine the transient head observed in the actual scenario, a comparison between the computed and the experimental results was done and is presented in Fig. 7. From the Fig. 7, one can see a huge difference between the computed transient results and experimental results. The discrepancy so observed indicates the occurrence of strong energy dissipation in the pipelines. A strong damping of the pressure wave is observed in the experimental results. It may be seen that the amplitude of experimental pressure curve decreases very quickly and the wave front get smoothened. It is found that, the pipes will be more prone to wave damping than that of computed using the classical water hammer equations with convective terms. This may be due to the fact that the classical water hammer equation cannot take into account the pipe wall elasticity and the subsequent energy dissipative mechanisms. This facilitates the need for the use of a different approach for analysing transient heads accurately.

B. Validating the observations using Surge2000

In order to check the accuracy of the modified base model, transient analysis for the system was even carried out using Surge2000 software. In Fig. 8, a comparison was done between the transient analysis results from the modified base model and that obtained from the software. The results obtained from the model shows more damping compared with the software results. This may be due to the addition of convective term in the modified base model. However, both the results show huge discrepancy from the actual experimental data.

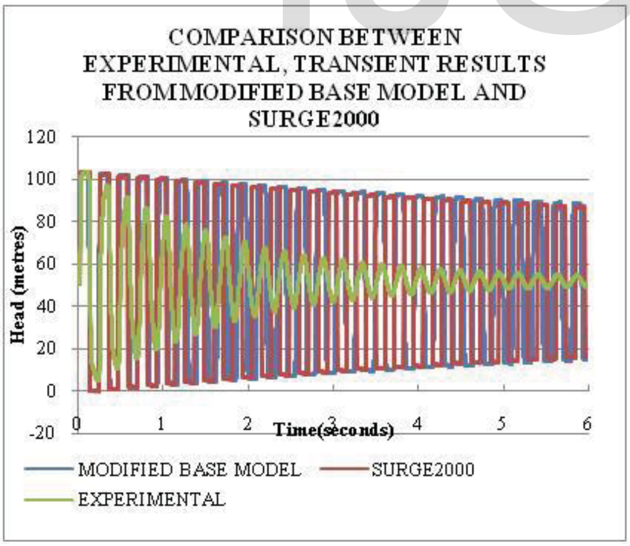

Fig. 8. Comparison between experimental results and transient results at the valve end from modified base model and from Surge2000

C. Incorporation of variable wave celerity into the continuity equation and diffusion term in the momentum equation

For increasing the ability of the model to predict the head variations accurately with the experimental data, modifications have to be done to the basic water hammer equations. The idea proposed in [11] was to relate the elastic behaviour of both the water and the wall of the pipe to the continuity equation via the variable pressure wave celerity. Also, the diffusive term was incorporated into the momentum equation to induce smoothing characteristics of the water hammer wave.The same approach is used in this study.

In the derivation of classical Joukovsky (Korteweg) formula for pressure increment (6), perfect elastic behaviour of the liquid and pipe material were assumed [15]. If real bodies are considered, during the compression of liquid in the pipe due to immediate valve closure, a fraction of energy, denoted as E_{diss}, is lost because of dissipation. Assuming that the dissipated energy constitutes a part of the elastic energy of the liquid and the pipe wall, this fraction can be denoted using the coefficient β [11].

The energy dissipation coefficient (β) was calculated using (11). The value of β at the first time state was obtained as 0.0843.The variable wave celerity was then calculated for each Δt. The corresponding variation in head at the valve is plotted in Fig. 9. Spurious oscillations are observed and the program collapses. The computed wave celerity at the valve end is provided in Fig. 10. The wave celerity ranged from 1489.888m/s to 1067.35 m/s with the median as 1220.707 m/s. The energy dissipation coefficient versus time is also plotted shown in Fig. 11. The mean value of β was obtained as 0.056898. The global diffusion coefficient (v^G) was taken as $600 m^2/s$ [11]. Equations (12) and (14) were taken as the governing equations for the modified model.

It is found that, the computed pressure head goes on increasing instead of reducing and attaining a stable value at the end of simulation time. This may be the result of numerical dispersion in the MacCormack scheme [1]. But, these dispersive errors cause high-frequency oscillations near steep gradient. Thus, the developed model would have to be revised to reduce these numerical dispersive errors.

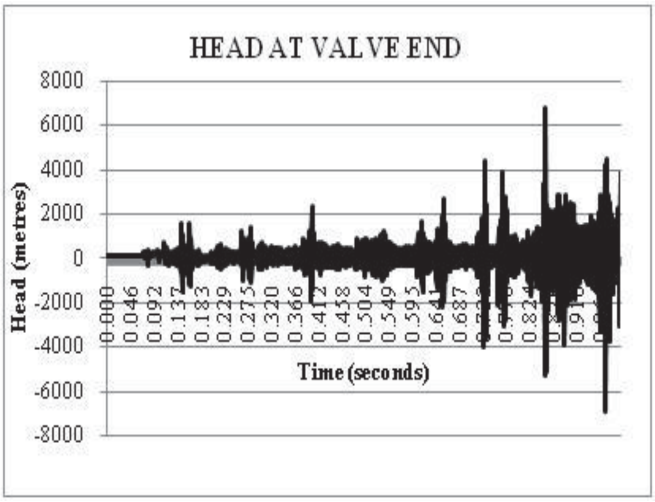

Fig. 9 Head variation at the valve end

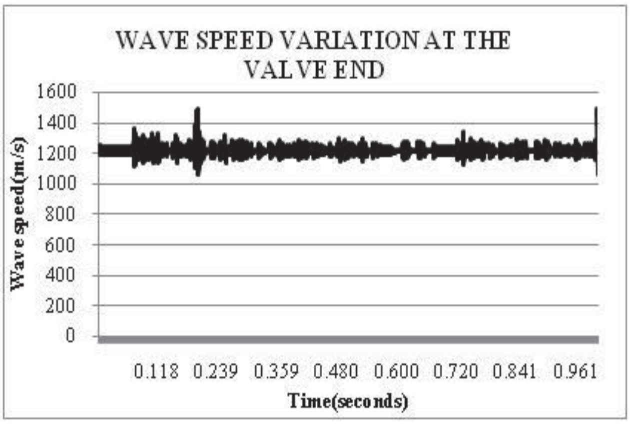

Fig. 10 Wave speed variation at the valve end

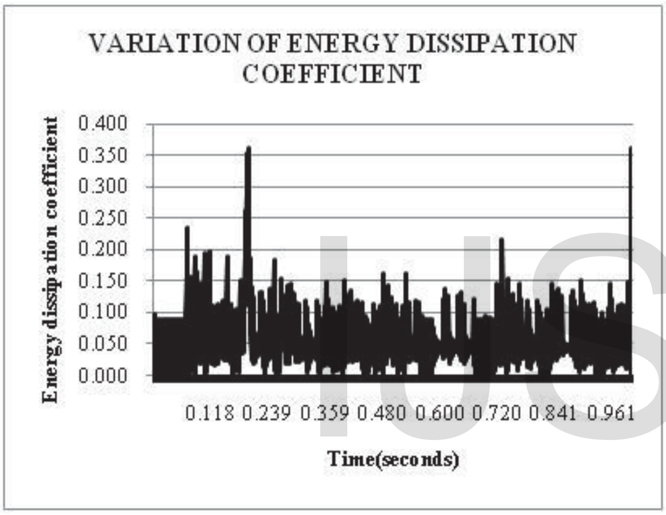

Fig. 11 . Variation of energy dissipation coefficient, β along the time axis

D. Modifying the existing model to reduce the dispersive errors

In order to remove the dispersive errors, a procedure developed by Chaudhry [5] was used to dampen the numerical instabilities. This procedure smoothes regions of large gradients while leaving smooth areas relatively undisturbed [1].

The method for removing the numerical dispersive error was carried out and the value of dissipation constant to regulate the amount of artificial viscosity was taken as 6 corresponding to the numerical diffusion coefficient as $600m^2/s$ [11]. The energy dissipation coefficient was taken as 0.05 corresponding to the mean obtained in the computed results. The obtained results are provided in Fig. 12.

From the Fig.12, it is clear that the head peak goes on decreasing after the time of valve closure and attains a stable head at some later point of time. It is clear from the figure that the stable head is obtained at the time of six seconds. For comparison, the computed results by the proposed method were superimposed on the experimental result values and the computed result values. In contrast to the transient results obtained earlier from the modified base model and that from software (Fig. 8), the computed results (Fig. 13) from the proposed method show good agreement with the experimental results. Although the results have numerical oscillations, the peak of the head is approximately equal to that of the experimental value.

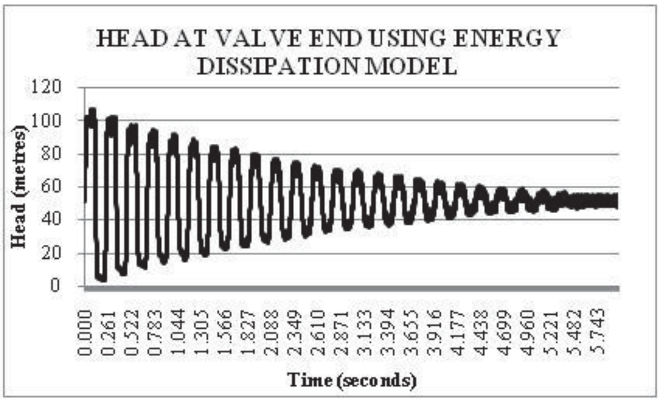

Fig. 12 Head at valve end using energy dissipation model

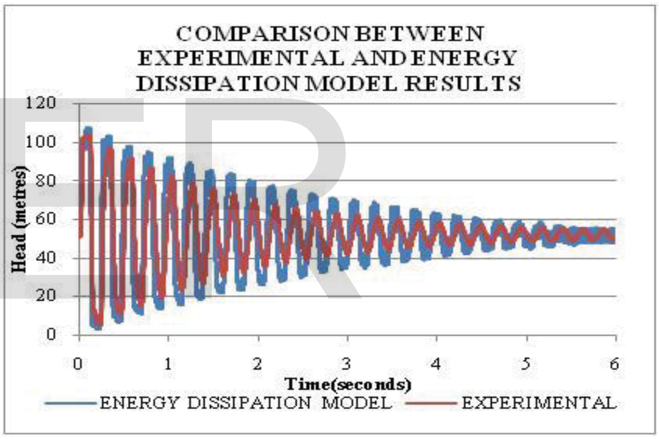

Fig. 13. Comparison between experimental and energy dissipation model

E. Error analysis

Assuming the experimental results to be true representation of the actual scenario, absolute mean percentage error was done for each computation and the results obtained are presented in Table II. For the steel pipe, transient analysis results from the modified base model showed lesser error compared to the Surge2000 software results. The proposed method showed 13.91% error in computation from the actual value.

TABLE II. ERROR ANALYSIS RESULTS

Material	**Method of Computation**	**Absolute Mean Percentage Error**
STEEL	Transient from modified base model	61.66%
	Transient from Surge2000	73.69%
	Proposed Method	13.91%

VIII. CONCLUSIONS

The current paper focuses on creating an appropriate model for energy dissipation during transient flow. The modified base model results were compared with the analysis results from the Surge2000 software for validation. It was found that the modified base model results damped faster than that computed from the software. Thus, it was found that the convective terms in the water hammer equations, generally neglected in the transient analysis were found necessary to better analyse the piping system. The equations were modified for incorporating variable wave celerity into the continuity equation and the diffusive term into the momentum equation. In the results obtained, large oscillation appeared and its magnitude increased with time. It was clear from the results that numerical dispersive errors occurred. Once the dispersive error was eliminated by introducing the artificial viscosity a better result was obtained in the present study owing to the fact that MacCormack scheme is dispersive in nature although dissipation-free.

An error analysis was carried out for the computation. It was found that the developed energy dissipation model gave better results than the transient analysis results from the Surge2000 software and classical waterhammer equations. The developed model not only predicts the first pressure hikes, but also the subsequent pressure peaks towards the stable values at the end of simulation. Thus, the developed model was able to predict the energy dissipation mechanism in a better way during transient.

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